Value at Risk and Conditional Value at Risk: A Comparison

Value at risk is praised as a simple, universal risk measure on the one hand and frequently referred to as "controversial" or "hotly debated" on the other. This article examines two key limitations of the measure, inability to quantify tail risk and lack of subadditivity, and considers whether a related measure, conditional value at risk, can overcome these limitations.

"Not everything that can be counted counts, and not everything that counts can be counted."
—Attributed to Albert Einstein

Raised for its ability to aggregate the risks of such disparate assets as equities, bonds, commodities, currencies, and options, value at risk (VaR) has become an industry standard in the world of risk measurement since its introduction in the early 1990s. Unlike most widely used risk measures, which are based on historical returns, VaR is a forward-looking measure of risk for estimating future portfolio losses. Used for calculating regulatory capital, VaR also serves as a benchmark measure, a potential loss measure, and a way to calculate equity capital when setting a firm's capital cushion. VaR's advantages are well known: It can aggregate various related and unrelated risks of a portfolio or firm into one universal measure expressed in currency terms. Despite its popularity, the measure remains the subject of much controversy and is blamed for contributing to substantial losses during the global financial crisis. This article explores some of VaR's most criticized aspects and considers whether a related measure, conditional value at risk, can address them.

VAR'S MAJOR LIMITATIONS

Value at risk is a statistical measure of the amount of money a portfolio, strategy, or firm might expect to lose over a specified time horizon with a given probability (usually 90%, 95%, or 99%). For example, a portfolio that is expected to lose no more than $1 million 95% of the time (or 19 of every 20 days) has a VaR of $1 million. On the downside, 5% of the time, or 1 day out of every 20, the portfolio is expected to lose at least $1 million. One of VaR's major criticisms is that it provides no information about how much the portfolio could lose (beyond the $1 million) during this 5% of the time.

Compare the above portfolio, Portfolio A, with a second portfolio, Portfolio B. Assume that when Portfolio A loses more than $1 million, its losses range up to $1.5 million. Portfolio B also has a VaR of $1 million at the 95% confidence level. When Portfolio B's losses exceed its VaR, however, it can lose as much as $5 million in a single day. Despite their very different risk profiles, the two portfolios have the same VaR. VaR ignores the total amount at risk in the left tail, or the risk of large, even catastrophic, loss.

A popular (but not the only) method of estimating VaR relies on the assumption of a symmetrical return distribution, which draws much criticism in a world full of investments with nonlinear risks, such as options, credit, and derivatives. This approach, called the analytical method, calculates a parametric VaR based on estimating returns, variances, and correlations for a portfolio's assets. Two other industry standard methods of estimating VaR are better suited to address a portfolio of assets with nonlinear risk assumptions: historical simulation and Monte Carlo simulation.

The historical, or nonparametric, approach can account for fat tails and skewness in distributions if they have been represented in past return patterns; prices can be weighted to emphasize more-recent movements. This approach, naturally, assumes future prices will behave as past prices have. The Monte Carlo simulation approach is more flexible, allowing analysts to specify myriad and complex risk factors,
and uses random simulations to generate returns within a given distribution. The distribution used in simulating can be of a variety of forms—from Gaussian bell shaped to the family of extreme value distributions. The flexibility gained in this approach comes at the expense of introducing greater model risk. Not surprisingly, the three different calculation methods can yield very different estimates.

A second major criticism of VaR is that the measure is not subadditive. Subadditivity is based on the principle of diversification. Subadditivity holds that adding the risk of Asset A and the risk of Asset B will not result in an overall risk that is greater than the sum of the two risks together. Jorion (2007), author of Value at Risk: The New Benchmark for Managing Financial Risk, describes subadditivity this way: “Merging two portfolios cannot increase risk” (p. 114). Because two individual assets can have a lower VaR than a portfolio of both assets combined, VaR violates this principle. Jorion uses short option positions as an exaggerated example. Lleo (2010) gives the example of a bank risk manager aggregating the VaR of various trading desks to obtain an overall VaR for trading operations. Because VaR lacks subadditivity, the manager cannot be sure the overall VaR realistically reflects diversification among the trading desks. Albanese (1997) shows that because of VaR's lack of subadditivity, using VaR in managing credit portfolio risk can lead to a higher concentration of credit risk. Credit instruments tend to have asymmetrical return profiles associated with default risk, and their return distributions can be fat tailed.

**CONDITIONAL VALUE AT RISK: A SUPERIOR RISK MEASURE?**

Many investors who lost significant amounts of money in the global financial crisis trace their failures to an improper understanding and use of VaR. Some admit knowingly turning a blind eye to tail risk, and others say they were caught off guard by the measure’s limitations. A number of these investors have since adopted a related risk measure, conditional value at risk (CVaR). Designed to measure the risk of extreme losses, CVaR is an extension of VaR that gives the total amount of loss given a loss event. While VaR users might ask, “How often could my portfolio lose at least $1 million?,” CVaR users would ask, “When my portfolio loses more than $1 million, how much could it lose?” CVaR is calculated by taking a weighted average of the VaR estimate and the expected losses beyond VaR. CVaR is calculated as a portfolio’s VaR plus the probability-weighted average loss expected in excess of VaR. A CVaR estimate cannot be lower than a VaR estimate. The relationship between VaR and CVaR is illustrated in the following graph:

![Figure 1. Conditional Value at Risk in Terms of the Probability Density Function](image)

Notes: The shaded area represents the losses that exceed the VaR. Cumulative probability in the shaded area is equal to \( \alpha \). Cumulative probability in the white area is equal to the confidence level, \( 1 - \alpha \).

CVaR is superior to VaR because CVaR quantifies tail risk and has been shown to be subadditive. CVaR can capture the minimal probability of a substantial loss for a strategy with an asymmetrical risk profile, such as for writing options. In contrast, the VaR for such a strategy would be artificially low and fail to reflect the potential magnitude of losses (Jorion 2007). CVaR has other advantages as well. Rockafellar and Uryasev (2000) show that when portfolio losses are estimated using a nonparametric method, portfolio risk is more easily optimized by using CVaR than VaR.

Amelia Hopkins, CFA, senior vice president at Granville Capital, says she supplements VaR estimates with CVaR when evaluating hedge fund managers: “In August [2011], I saw many managers blow right through their monthly VaR at the 99% significance level—and then have a repeat performance in September,” Hopkins says. “How do you explain a 3 to 4 standard deviation event occurring in two consecutive months? It is clearly a deficiency of returns-based analysis and causes me to crave a better model of downside risk. . . . CVaR may not be the Holy Grail, but it is widely used.”

Like any risk measure, CVaR has its shortcomings. VaR estimates tend to be more stable than CVaR estimates for the same confidence level; CVaR often requires a large number of observations to generate a reliable estimate, and it is more sensitive to estimation errors than VaR (Yamai and Yoshiha 2002). Moreover, the reliability of CVaR depends substantially on the accuracy of the tail model used. Investors also should understand that because CVaR is based on an average loss beyond VaR, it is not a measure of the most extreme potential loss.

HOW DO THEY COMPARE?

Conditional value at risk is subadditive, accounts for tail risk, and can be used to optimize portfolios for both VaR and CVaR. Despite CVaR’s superior mathematical properties, there is plenty of support for VaR. Don Ross, CFA, founder and chief investment officer of Alpha Squared Capital Management, LLC, manages an ETF-based global macro fund and is a longtime VaR user. Ross uses VaR as a benchmark for relative volatility. He says, “We invest across asset classes—any and all asset classes, long–short, leveraged or not. VaR boils the risk down to one measure.” Ross adds that, although he does not directly invest in options or derivatives, he supplements VaR with stress testing and prefers both the historical approach and the Monte Carlo simulation to the analytical method of estimating VaR. He generally calculates VaR according to all three methods and compares them. “Almost always,” he says, “the parametric method shows the lowest volatility. The Monte Carlo method seems to be the most useful.”

Jorion is perhaps VaR’s best known supporter. He points out that VaR was developed for normal return distributions under normal market conditions—that is, markets that function as expected (i.e., no liquidity crisis). But this does not necessarily rule out VaR’s usefulness for a portfolio containing assets with asymmetrical payoffs. In “In Defense of VaR,” Jorion (1997) states, “A symmetric, normal approximation may be appropriate for large portfolios, in which independent sources of risk, by the law of large numbers, tend to create normal distributions.” For option-heavy portfolios or those with undiversified credit risk, however, he recommends implementations of VaR that provide for asymmetrical return profiles rather than the parametric approach.

What about the charge that VaR lacks subadditivity? For normally distributed returns, Jorion (2007) notes that VaR is, in fact, subadditive. VaR was originally developed to measure market risk, or the risks associated with interest rates, currency fluctuations, and asset prices. When measuring market risk, VaR is also generally found to be subadditive. VaR does, however, lack subadditivity when applied to credit risk. CVaR can model the asymmetrical risk of credit default better than VaR.

Finally, it is essential to routinely backtest and stress-test VaR estimates. Backtesting, the process of evaluating how frequently actual losses exceed VaR estimates, is critical to determining how well the model functions. Stress testing using stylized scenarios, actual extreme events, or hypothetical events can provide information about potential fat tails and extreme losses.

This article provides only a brief overview of the considerations associated with using value at risk and conditional value at risk. Much has been written about VaR, and its pros and cons have been extensively debated. Regulatory requirements aside, the two measures tend to appeal to different types of users and/or for different evaluation purposes. A trader using VaR as a potential loss measure may prefer VaR over CVaR because VaR is less restrictive; the firm’s owner may prefer CVaR because it is more conservative (Uryasev et al. 2010). VaR may be
preferred when a reliable tail model is not available. CVaR may be preferred when a more conservative equity capital cushion is desired. VaR and CVaR are also frequently used as complements to each other. Overall, CVaR can help address VaR’s indifference to tail losses and lack of subadditivity, but one major limitation it cannot address is lack of knowledge by the VaR user. Ross says, “I rely on value at risk,” although, he adds, “I understand the VaR limitations.”

**Notes**

1. This approach is also referred to as the “delta-normal method” and the “variance-covariance method.”
2. The term “CVaR” is sometimes used interchangeably with “expected shortfall,” although there are slight differences between the two. See Lleo (2010) for an explanation. In addition, CVaR is also known as “tail VaR” and “expected tail loss.”

**Resources**


**Disclosure:** Don Ross provides advisory services to Boyd Watterson Asset Management.

Deborah Kidd, CFA, is senior vice president at Boyd Watterson Asset Management, LLC.